Solutions - Midterm Exam

(February 16th @ 5:30 pm)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (20 PTS)

a) Complete the following table. Use the fewest number of bits in each case: (9 pts.)

REPRESENTATION			
Decimal	Sign-and-magnitude	1's complement	2's complement
-89	11011001	10100110	10100111
37	0100101	0100101	0100101
-61	1111101	1000010	1000011
-20	110100	101011	101100
-64	11000000	10111111	1000000

b) Convert the following decimal numbers to their 2's complement representations. (3 pts.)

- ✓ -16.1875
  - +16.1875 = 010000.0011
  - ⇒ 101111.1101
- ✓ 37.3125
  - +37.3125 = 0100101.0101

c) Perform the following operations, where numbers are represented in 2's complement. For each case, use the fewest number of bits to represent the summands and the result so that overflow is avoided. (8 pts)

- ✓ -89 + 128
- ✓ -61 -13

n = 9 bits

$c_9=1$, $c_8=1$, $c_7=0$, $c_6=0$, $c_5=0$, $c_4=0$, $c_3=0$, $c_2=0$, $c_1=0$, $c_0=0$

$c_9 \oplus c_8 = 0$
No Overflow

```
 -89 = 1 1 0 1 0 0 1 1 1 +
 128 = 0 1 0 0 0 0 0 0 0
 ------------------------
  39 = 0 0 0 1 0 0 1 1 1
```

$-89 + 128 = 39 \in [-2^8, 2^8-1] \rightarrow$ no overflow

n = 7 bits

$c_7=1$, $c_6=0$, $c_5=0$, $c_4=0$, $c_3=0$, $c_2=1$, $c_1=1$, $c_0=0$

$c_7 \oplus c_6 = 1$
Overflow!

```
 -61 = 1 0 0 0 0 1 1 +
 -13 = 1 1 1 0 0 1 1
 --------------------
       0 1 1 0 1 1 0
```

$-61 -13 = -74 \notin [-2^6, 2^6-1] \rightarrow$ overflow!

To avoid overflow: n = 8 bits (sign-extension)

$c_8=1$, $c_7=1$, $c_6=0$, $c_5=0$, $c_4=0$, $c_3=0$, $c_2=1$, $c_1=1$, $c_0=0$

$c_9 \oplus c_8 = 0$
No Overflow

```
 -61 = 1 1 0 0 0 0 1 1 +
 -13 = 1 1 1 1 0 0 1 1
 ----------------------
 -74 = 1 0 1 1 0 1 1 0
```

$-61 -13 = -74 \in [-2^7, 2^7-1] \rightarrow$ no overflow

PROBLEM 2 (18 PTS)

- Calculate the result of the following operations. The operands are signed fixed-point numbers. The result must be a signed fixed point number. For the division, use $x = 5$ fractional bits.

<table>
<tr><td colspan="3">1.0001 +
1.001001</td><td colspan="3">1000.0101 −
1.010101</td></tr>
<tr><td colspan="2">01.011 ×
1.01101</td><td colspan="2">1.001 ×
1.0101</td><td colspan="2">01.01110 ÷
1.011</td></tr>
</table>

```
c8=1 c7=1 c6=0 c5=0 c4=0 c3=0 c2=0 c1=0 c0=0
 1 1.0 0 0 1 0 0 +
 1 1.0 0 1 0 0 1
 ---------------
 1 0.0 0 1 1 0 1
```

```
                              c10=0 c9=0 c8=0 c7=0 c6=0 c5=0 c4=0 c3=0 c2=0 c1=0 c0=0
1 0 0 0.0 1 0 1 0 0 -         1 0 0 0.0 1 0 1 0 0 +
1 1 1 1.0 1 0 1 0 1     ➡     0 0 0 0.1 0 1 0 1 1
                              -------------------
                              1 0 0 0.1 1 1 1 1 1
```

```
 01.011 x  ➡  01.011 x  ➡      1 0 0 1 1 x
1.01101       0.10011            1 0 1 1
                               ---------
                               1 0 0 1 1
                             1 0 0 1 1
                         1 0 0 1 1
                       -----------------
                       0 1 1 0 1 0 0 0 1
                               ⬇
                       0.1 1 0 1 0 0 0 1
                               ⬇
                       1.0 0 1 0 1 1 1 1
```

```
1.001 x  ➡  0.111 x  ➡      1 0 1 1 x
1.0101      0.1011            1 1 1
                            -------
                            1 0 1 1
                          1 0 1 1
                        1 0 1 1
                      -------------
                      0 1 0 0 1 1 0 1
                            ⬇
                      0.1 0 0 1 1 0 1
```

✓ $\frac{01.01110}{1.011}$: To unsigned (denominator) and then alignment, $a = 4$: $\frac{01.0111}{0.101} = \frac{01.0111}{00.1010} = \frac{010111}{001010} \equiv \frac{10111}{1010}$

```
         0001001001
1010 ⟌ 1011100000
       1010
       ----
          1100
          1010
          ----
            10000
             1010
             ----
              110
```

Append $x = 5$ zeros: $\frac{1011100000}{1010}$

Integer Division:

$Q = 1001001, R = 110$
$\rightarrow Qf = 10.01001 \ (x = 5)$

Final result (2C): $\frac{01.01110}{1.011} = 2C(010.01001) = 101.10111$

PROBLEM 3 (26 PTS)

- Calculate the result (provide the 32-bit result) of the following operations with single floating point numbers. Truncate the results when required. When doing fixed-point division, use $x = 4$ fractional bits.

✓ 40B80000 − 42FA8000	✓ 80123000 + FACE8000	✓ 0A800000 × FAB80000	✓ 7B390000 ÷ C8C00000

✓ X = 40B80000 − 42FA8000:

40B80000: 0100 0000 1011 1000 0000 0000 0000 0000

$e + bias = 10000001 = 129 \rightarrow e = 129 - 127 = 2$ $\quad Mantissa = 1.0111$

40B00000 = 1.0111×2^2

42FA8000: 0100 0010 1111 1010 1000 0000 0000 0000

$e + bias = 10000101 = 133 \rightarrow e = 133 - 127 = 6$ $\quad Mantissa = 1.11110101$

42FA8000 = 1.11110101×2^6

$$X = 1.0111 \times 2^2 - 1.11110101 \times 2^6 = \frac{1.0111}{2^4} \times 2^6 - 1.11110101 \times 2^6 = (0.00010111 - 1.11110101) \times 2^6$$

To subtract these unsigned numbers, we first convert to 2C:

$R(2C) = 0.00010111 - 01.11110101 = 0.00010111 + 10.00001011$

The result in 2C is: $R = 10.00100010$, $-R = 01.11011110$

Floating point: we need the result in sign-and-magnitude: $\Rightarrow R(SM) = -1.11011110$

```
c10=0 c9=0 c8=0 c7=0 c6=0 c5=1 c4=1 c3=1 c2=1 c1=1 c0=0
 0 0.0 0 0 1 0 1 1 1 +
 1 0.0 0 0 0 1 0 1 1
 -------------------
 1 0.0 0 1 0 0 0 1 0
```

$X = -1.1101111 \times 2^6$, $e + bias = 6 + 127 = 133 = 10000101$

X = 1100 0010 1110 1111 0000 0000 0000 0000 = C2EF0000

✓ X = 80123000 + FACE8000:
80123000: 1000 0000 0001 0010 0011 0000 0000 0000
$e + bias = 00000000 = 0 \rightarrow$ *Denormal number* $\rightarrow e = -126$
80123000 = $-0.00100100011 \times 2^{-126}$

FACE8000: 1111 1010 1100 1110 1000 0000 0000 0000
$e + bias = 11110101 = 245 \rightarrow e = 245 - 127 = 118$
FACE8000 = $-1.10011101 \times 2^{118}$

$X = -0.00100100011 \times 2^{-126} - 1.10011101 \times 2^{118} = -\dfrac{0.00100100011}{2^{244}} \times 2^{118} - 1.10011101 \times 2^{118}$
The division by 2^{244} needs more than $p + 1 = 24$ bits for proper representation $\rightarrow$ we approximate this number by 0.
$X = -1.10011101 \times 2^{118}$
X = 1111 1010 1100 1110 1000 0000 0000 0000 = FACE8000

✓ X = 0A800000 × FAB80000:
0A800000: 0000 1010 1000 0000 0000 0000 0000 0000
$e + bias = 00010101 = 21 \rightarrow e = 21 - 127 = -106$ $\quad$ $Mantissa = 1.0$
0A800000 = 1.0×2^{-106}

FAB80000: 1111 1010 1011 1000 0000 0000 0000 0000
$e + bias = 11110101 = 245 \rightarrow e = 245 - 127 = 118$ $\quad$ $Mantissa = 1.0111$
FAB80000 = -1.0111×2^{118}

$X = (-1.0 \times 2^{-106}) \times (1.0111 \times 2^{118}) = -1.0111 \times 2^{12}$, $e + bias = 12 + 127 = 139 = 10001011$
X = 1100 0101 1011 1000 0000 0000 0000 0000 = C5B800000

✓ X = 7B390000 ÷ C8C00000:
7B390000: 0111 1011 0011 1001 0000 0000 0000 0000
$e + bias = 11110110 = 246 \rightarrow e = 246 - 127 = 119$ $\quad$ $Mantissa = 1.0111$
7B390000 = 1.0111001×2^{119}

C8C00000: 1100 1000 1100 0000 0000 0000 0000 0000
$e + bias = 10010001 = 145 \rightarrow e = 145 - 127 = 18$ $\quad$ $Mantissa = 1.1$
C8C00000 = -1.1×2^{18}

$X = -\dfrac{1.0111001 \times 2^{119}}{1.1 \times 2^{18}} = -\dfrac{1.0111001}{1.1} \times 2^{101}$

Alignment:
$\dfrac{1.0111001}{1.1} = \dfrac{1.0111001}{1.1000000} = \dfrac{10111001}{11000000}$

Append $x = 4$ zeros: $\dfrac{10111001\mathbf{0000}}{11000000}$

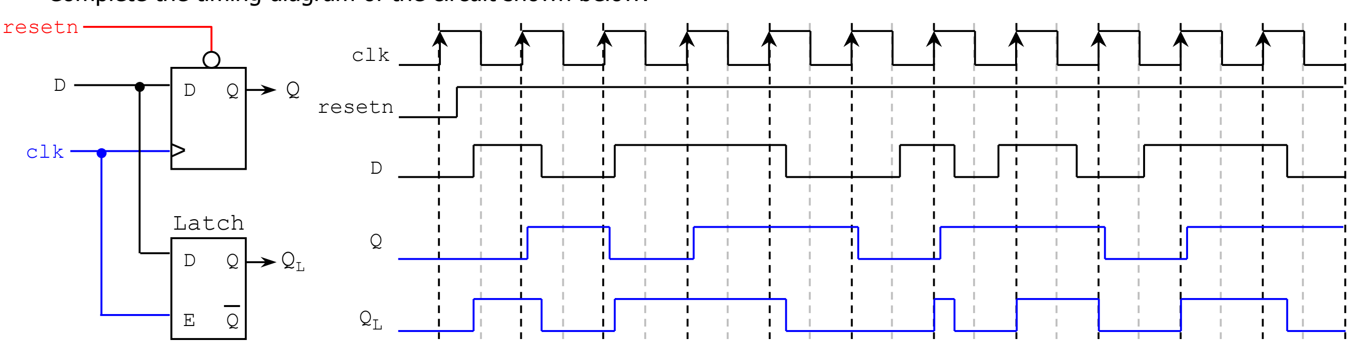
Integer division: $Q = 1111 \rightarrow Qf = 0.1111$

```
          000000001111
11000000 ) 101110010000
            11000000
            ---------
            101100100
             11000000
             ---------
             101001000
              11000000
              ---------
              100010000
               11000000
               ---------
                1010000
```

Thus: $X = -0.1111 \times 2^{101} = -1.111 \times 2^{100}$
$e + bias = 100 + 127 = 227 = 11100011$
X = 1111 0001 1111 0000 0000 0000 0000 0000 = F1F00000

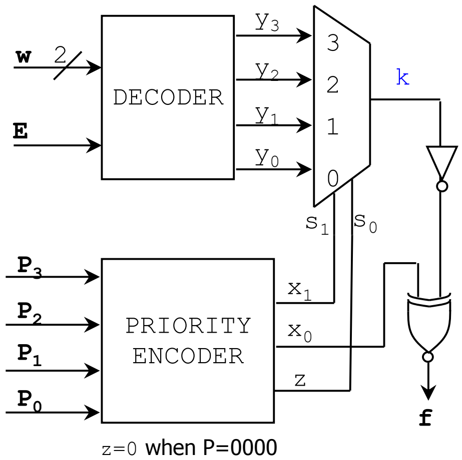
PROBLEM 4 (8 PTS)

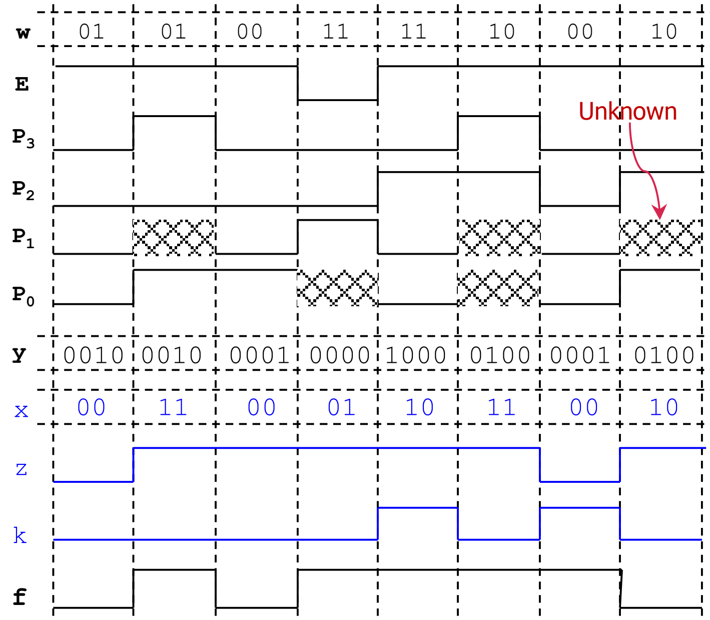
- Complete the timing diagram of the circuit shown below:



PROBLEM 5 (13 PTS)

- Complete the timing diagram of the circuit shown below. Note that $x = x_1x_0, y = y_3y_2y_1y_0$





PROBLEM 6 (15 PTS)

- The following circuit is a parallel/serial load shift register with enable input. Shifting operation: `s_l=0`. Parallel load: `s_l=1`.
  - ✓ Complete the timing diagram of the circuit.